Investigating the Dynamics of Canonical Flux Tubes

US-Japan Compact Toroid Workshop 8/24/2016

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LLNL-PRES-700761

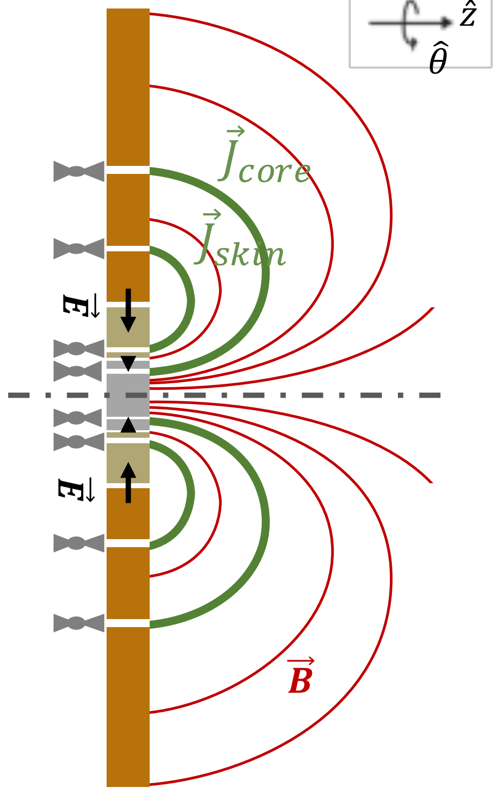
Mochi.LabJet - Study evolution of canonical flux tubes with core-skin currents and helical flows

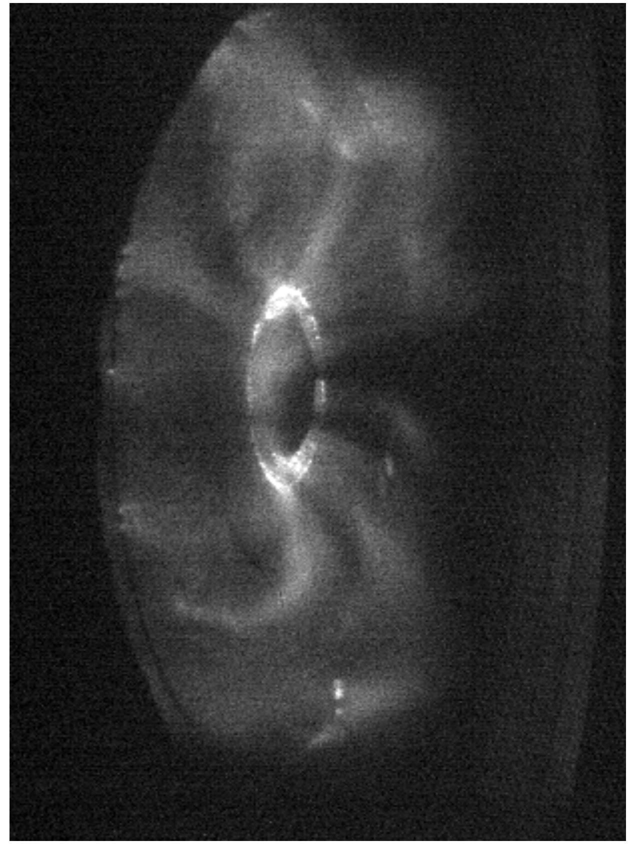
1. Study the stability of lengthening magnetic flux tubes with core and skin currents.

$$\psi = \int \vec{B} \cdot d\vec{S}$$

2. Reconstruct canonical flux tubes from magnetic field and ion flow measurements.

$$\Psi_\sigma = \int \vec{\Omega}_\sigma \cdot d\vec{S} = \int \left(m_\sigma \vec{\omega}_\sigma + q_\sigma \vec{B}\right) \cdot d\vec{S}$$





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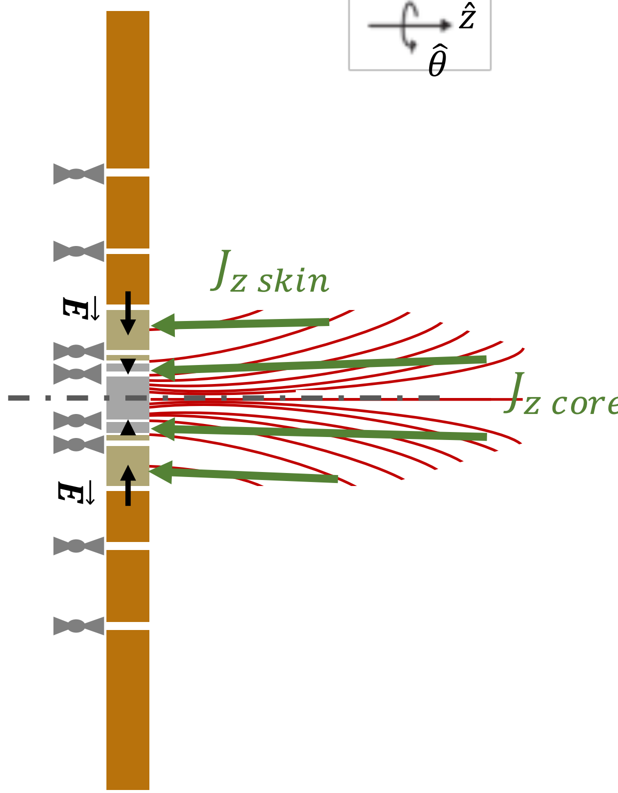

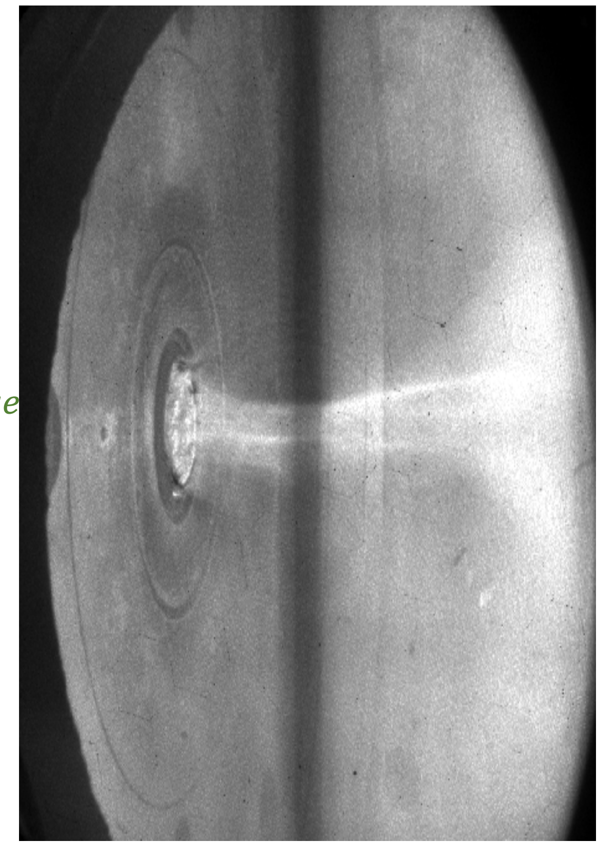

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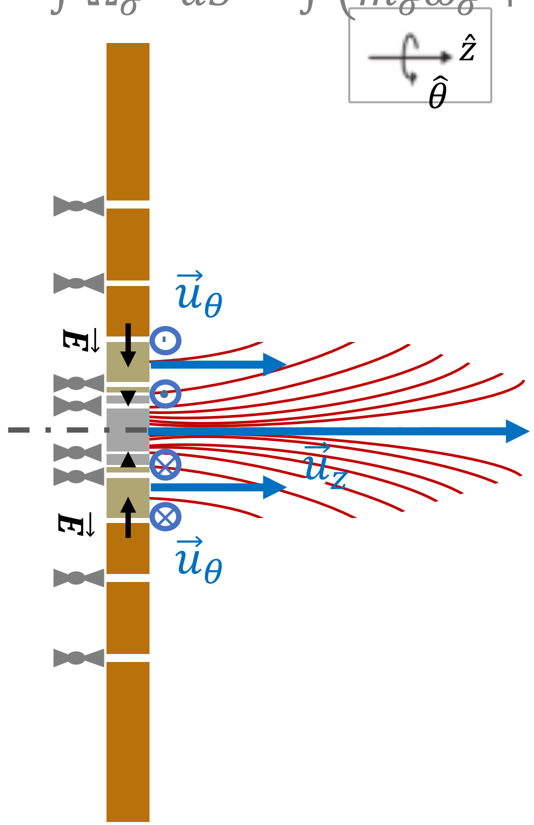

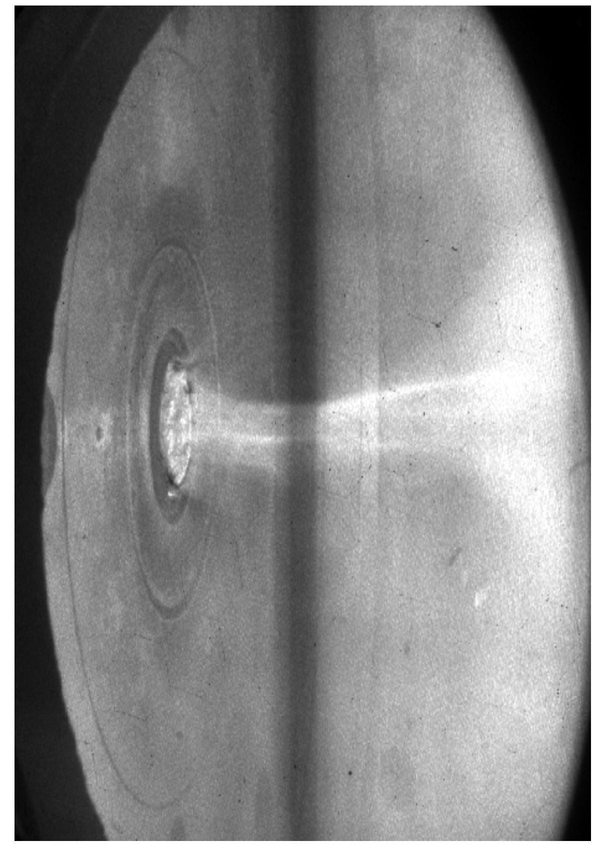

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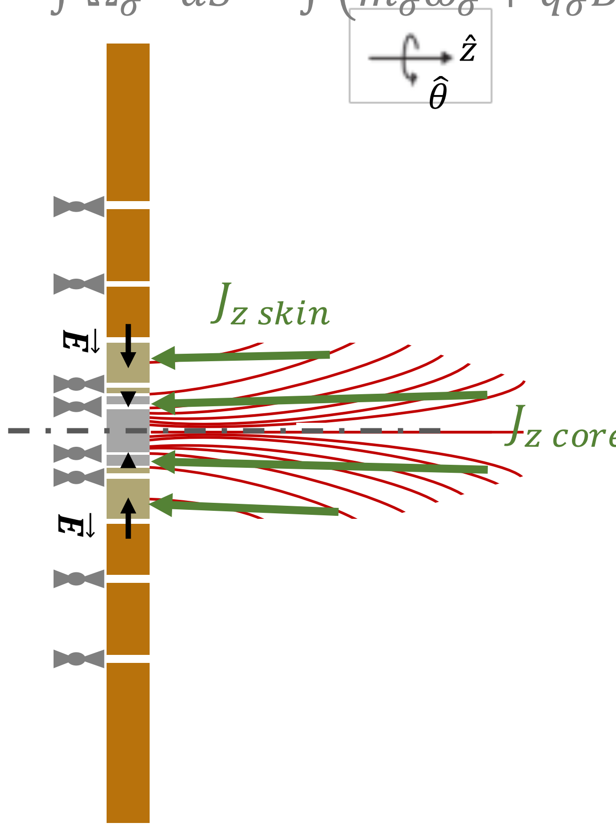
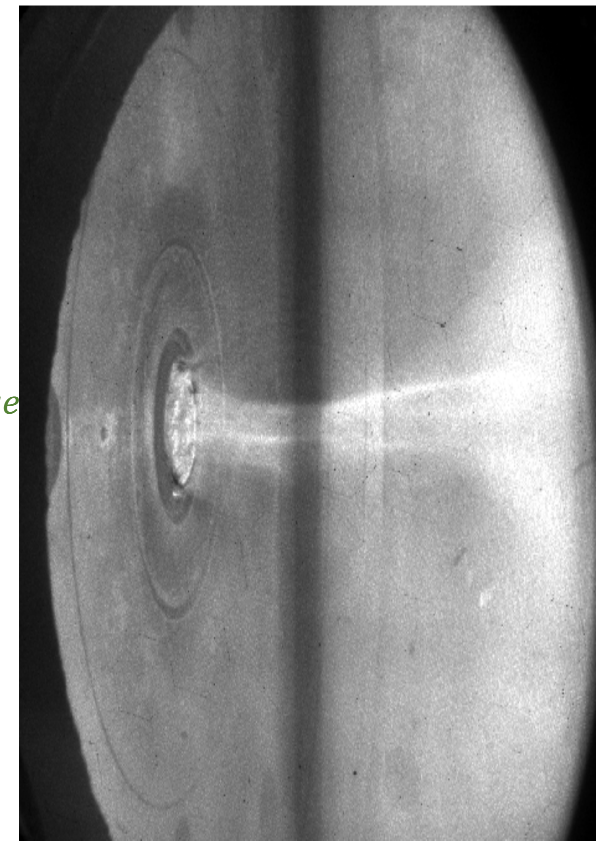
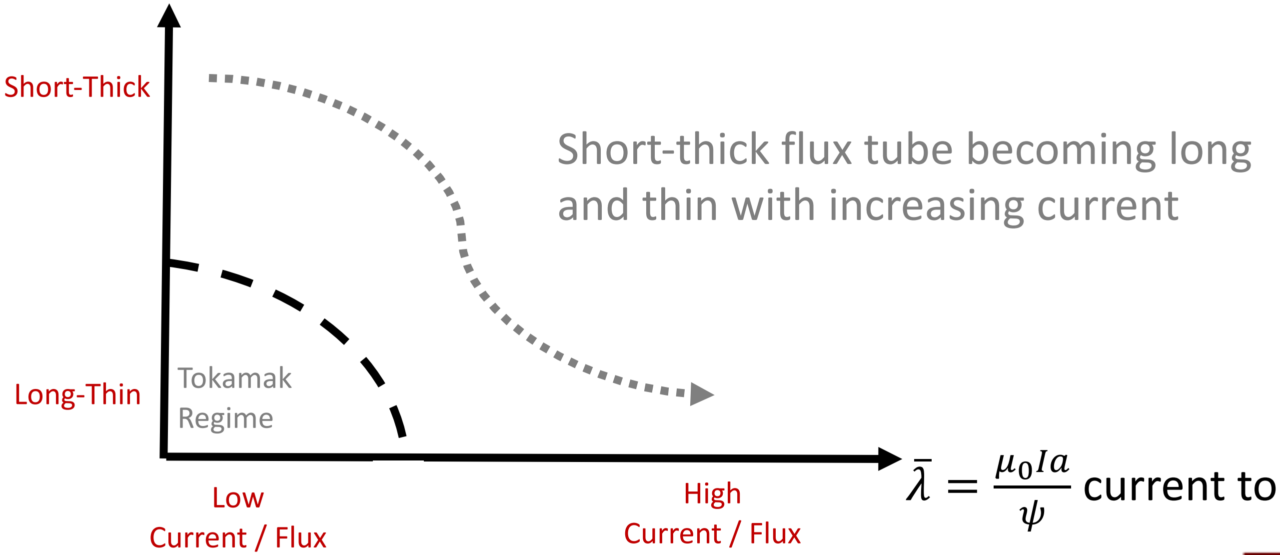


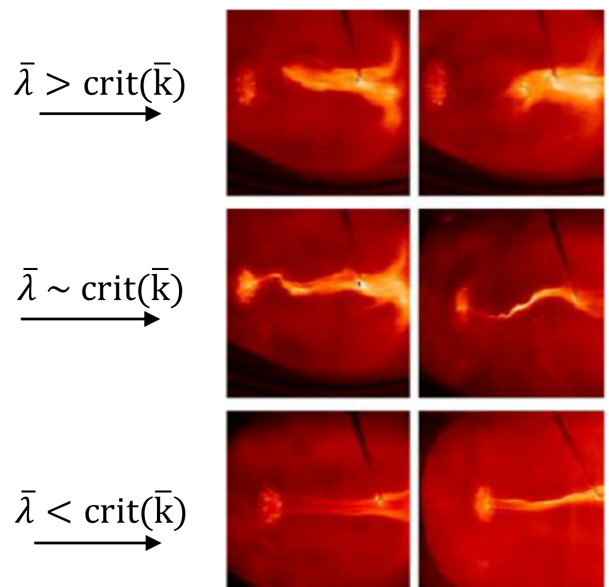


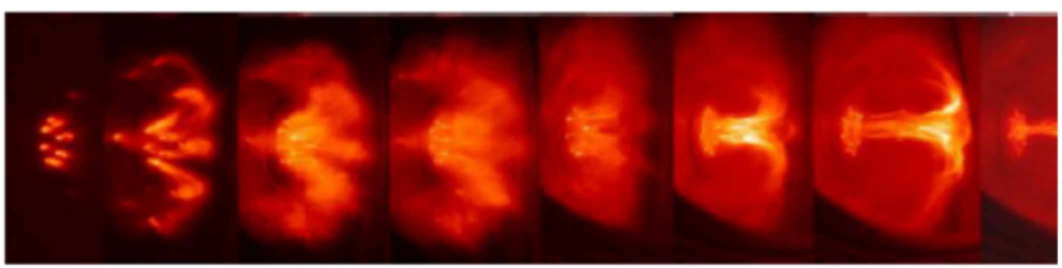
Define a $\bar{k} - \bar{\lambda}$ space for flux tubes

$\bar{k} = \frac{2\pi a}{L}$ inverse aspect ratio



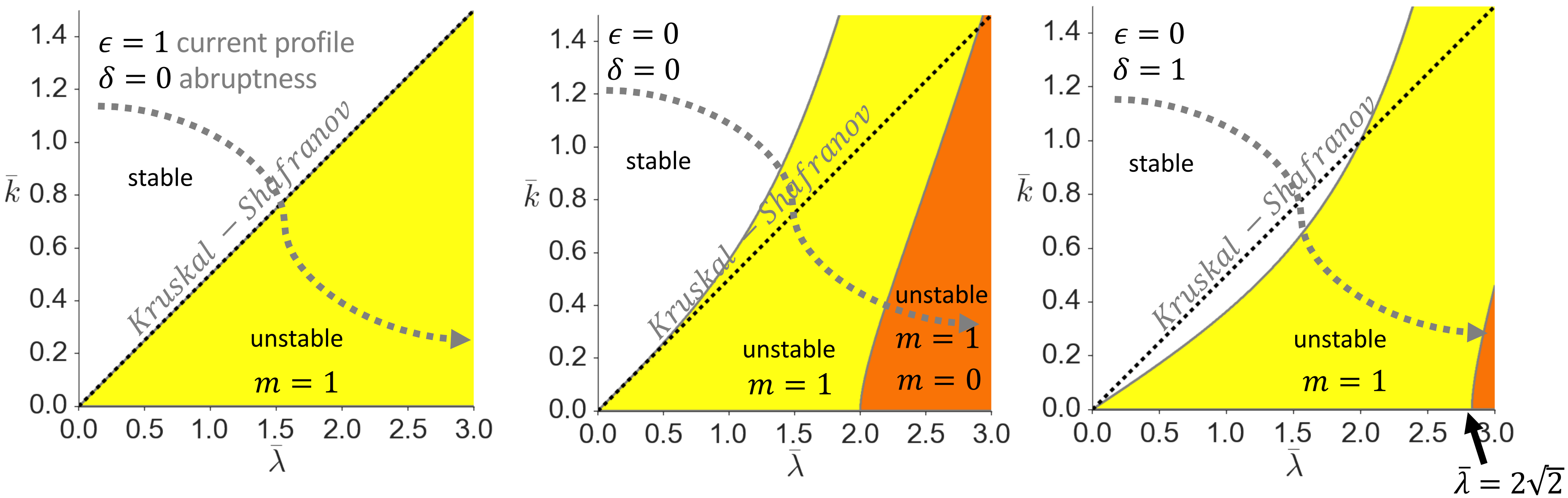
$\bar{\lambda} = \frac{\mu_0 I a}{\psi}$ current to magnetic flux ratio



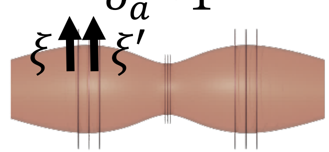


You, Yun & Bellan (2003)

Analytical $\bar{k} - \bar{\lambda}$ space: lengthening current-carrying flux tube crosses the sausage instability boundary



$m = 0$
$\delta_a \sim 1$



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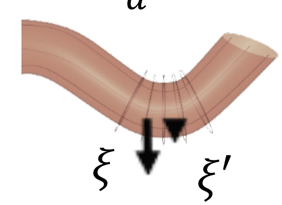
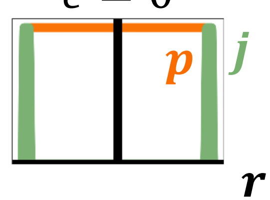
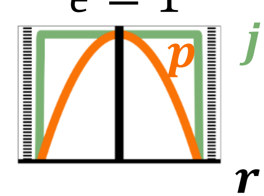


$\delta_a = \frac{\xi_a'}{\xi_a} a$ “abruptness”

$\epsilon = 0$



$\epsilon = 1$



$\epsilon = \frac{\bar{\lambda}_p}{\bar{\lambda}_v} = \frac{I_{core}}{I_{total}}$

Analytical $\bar{k} - \bar{\lambda}$ space: derived with textbook linear ideal MHD but with both core and skin currents

Simplify with Newcomb (1960) analysis of internal stability

Simplify with Bellan (2003) analysis of flared flux tubes

Set wall to ∞ Ignore wall effects

$$\delta W(\xi_r) = \delta W_{plas} + \delta W_{intf} + \delta W_{vac} > 0$$

$\Downarrow$

Stability Criterion

$$\frac{[2\bar{k} - m\epsilon\bar{\lambda}][(\delta + 1)2\bar{k} - (\delta - 1)m\epsilon\bar{\lambda}]}{\bar{k}^2 + m^2} + (\epsilon^2 - 1)\bar{\lambda}^2 - \frac{(m\bar{\lambda} - 2\bar{k})^2}{\bar{k}} \frac{K_m(|\bar{k}|)}{K'_m(|\bar{k}|)} > 0$$

δ can only be determined by integrating Euler-Lagrange equation

$$\delta W(\xi_r) = \delta W_{plas} + \delta W_{intf} + \delta W_{vac}$$

δW_{intf}: Simplify with Bellan (2003) analysis of flared flux tubes

δW_{vac}: Set wall to ∞ Ignore wall effects

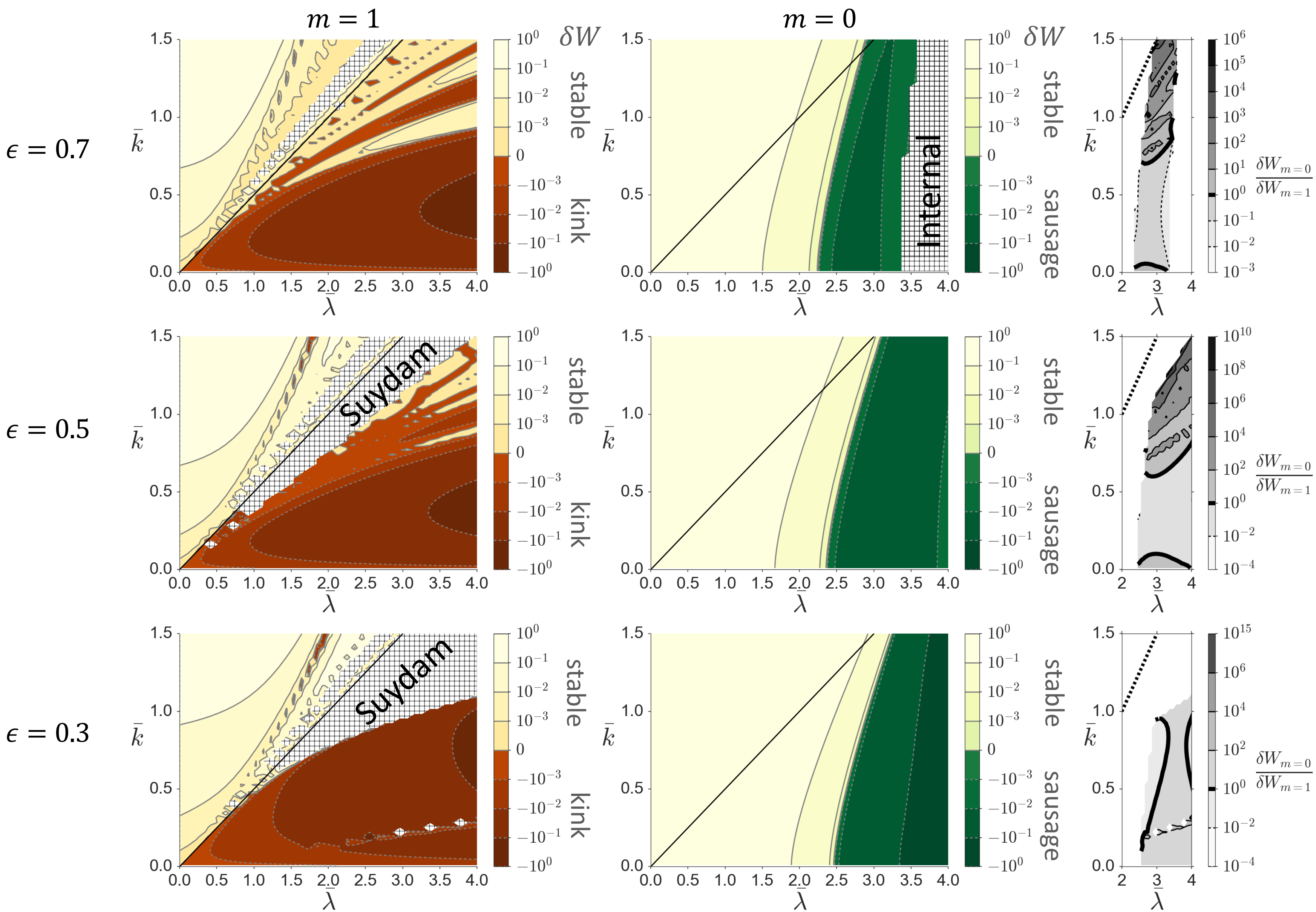
δW_{plas}: Simplify with Newcomb (1960) analysis of internal stability

$$\delta W_{plas} = \frac{\pi L}{2\mu_0}\int_0^a (f\xi'^2 + g\xi^2)dr + [h\xi^2]_0^a$$

Minimize δW_{plas} with solutions to $\text{Euler} - \text{Lagrange:}\ \frac{\partial}{\partial r}\left[f\frac{\partial \xi}{\partial r}\right] - g\xi = 0$

$$\Rightarrow \delta W_{plas} = \frac{\pi L \xi_a^2}{2\mu_0}\left[\frac{f_a}{a}\delta + h_a\right]$$

Numerical $\delta_a(\bar{\lambda}, \bar{k})$ results: current profile dependence and significant sausage unstable region in $\bar{\lambda} - \bar{k}$ space



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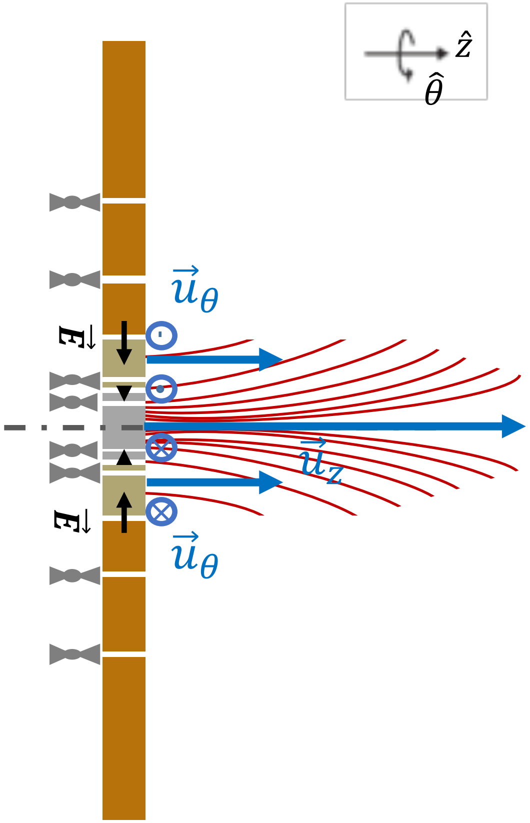
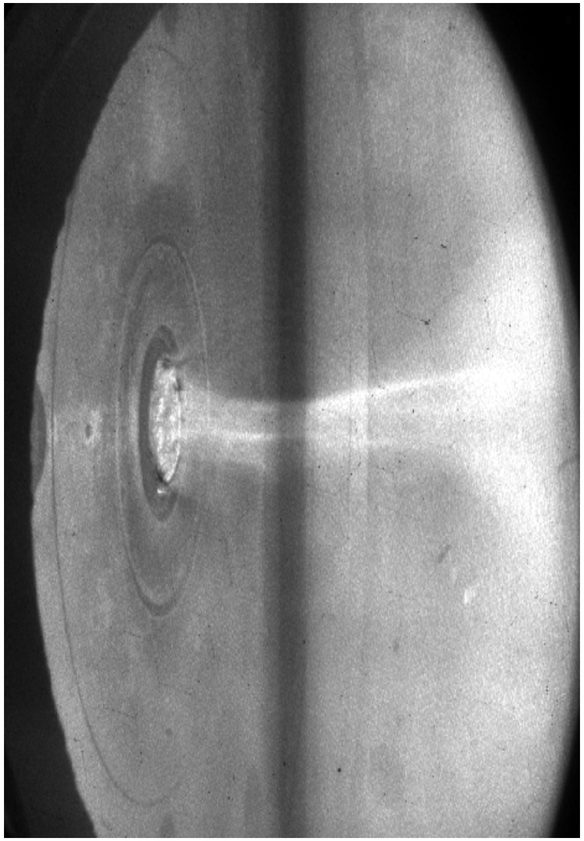
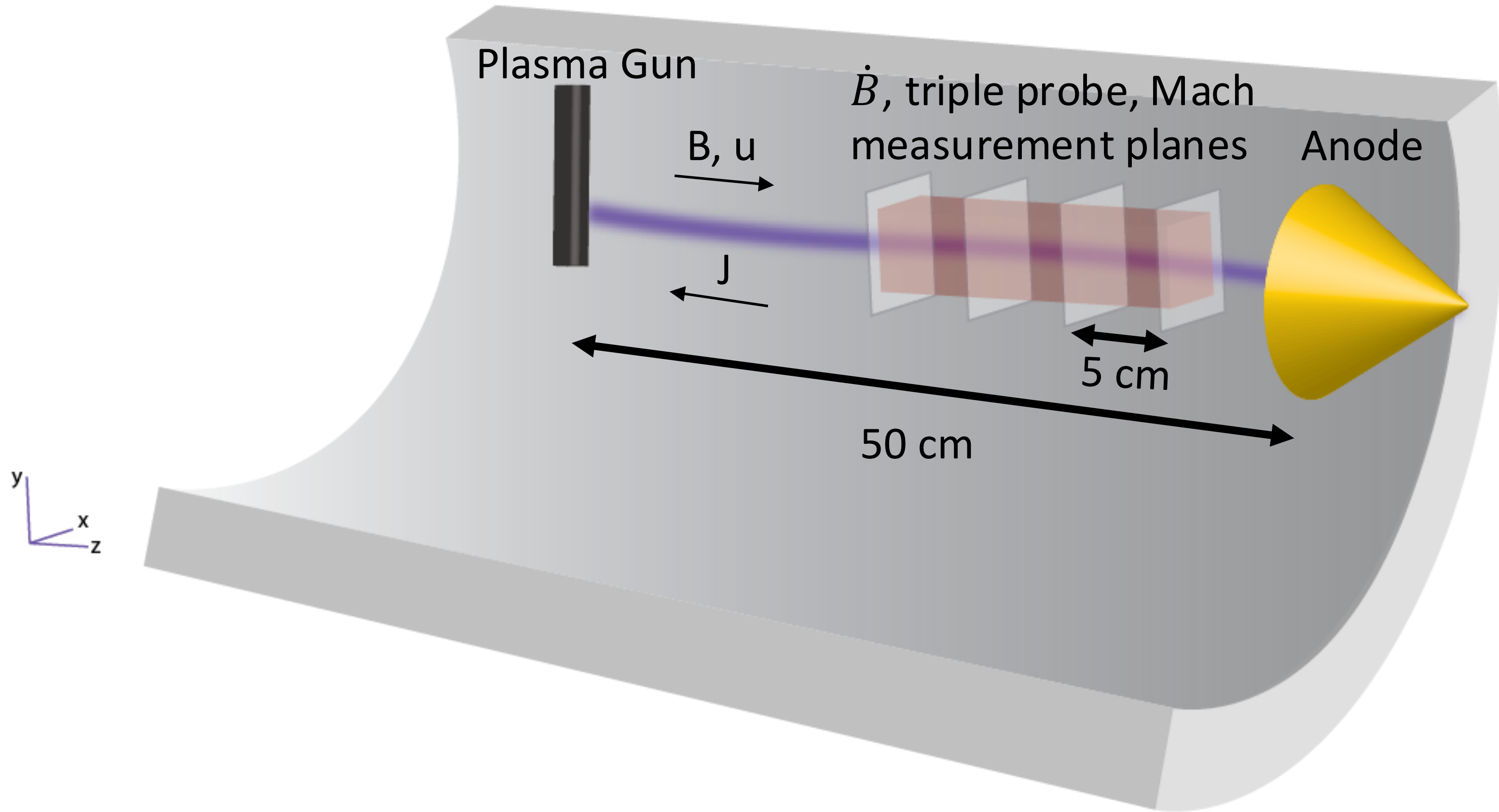
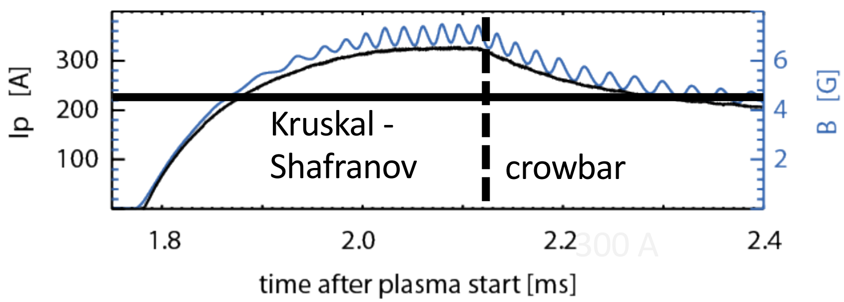




Reconstructing the RSX gyrating canonical flux tubes

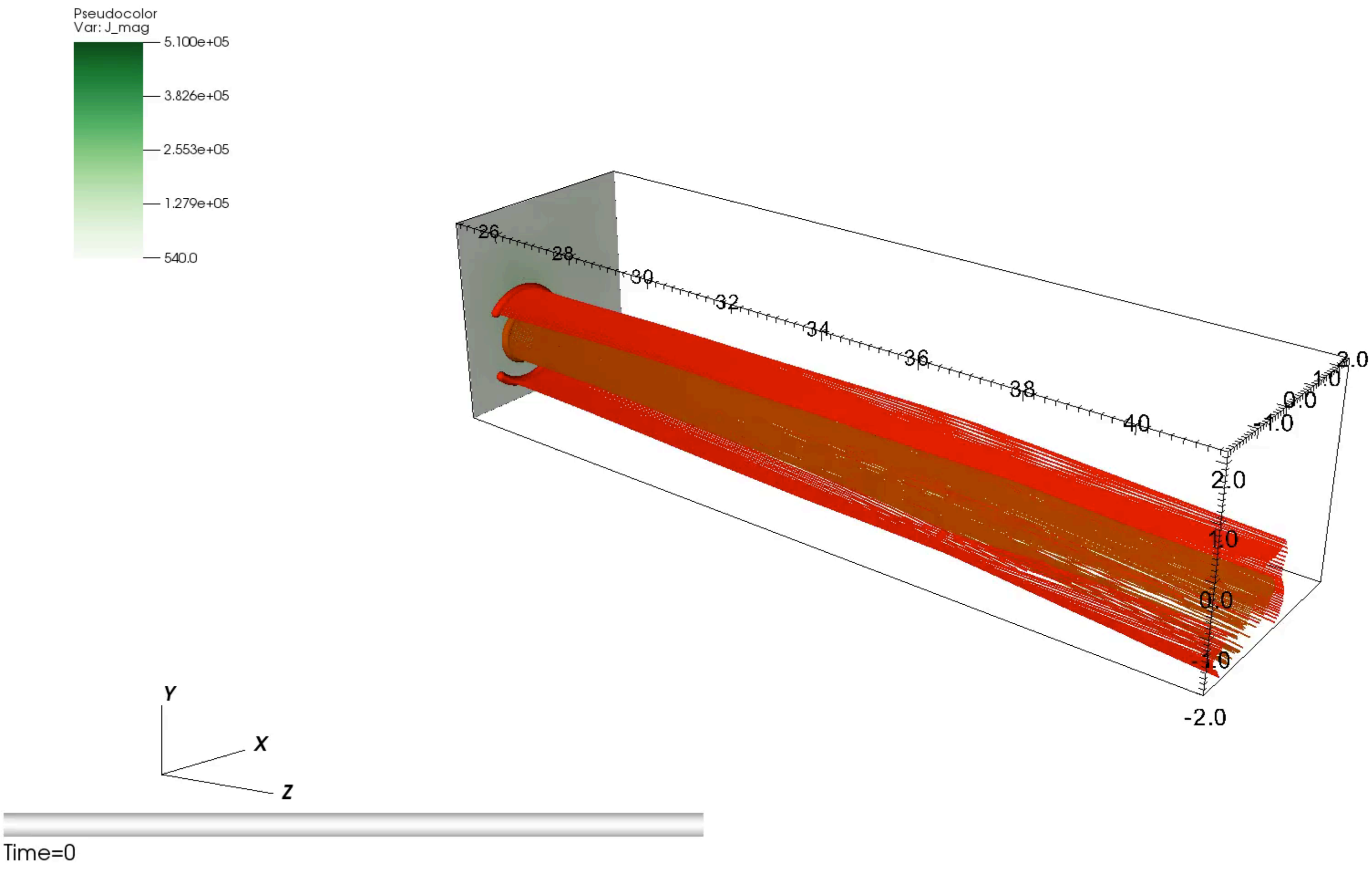




Gyration frequency is coherent across shots. Conditional sampling aligns traces from 3,000 shots.

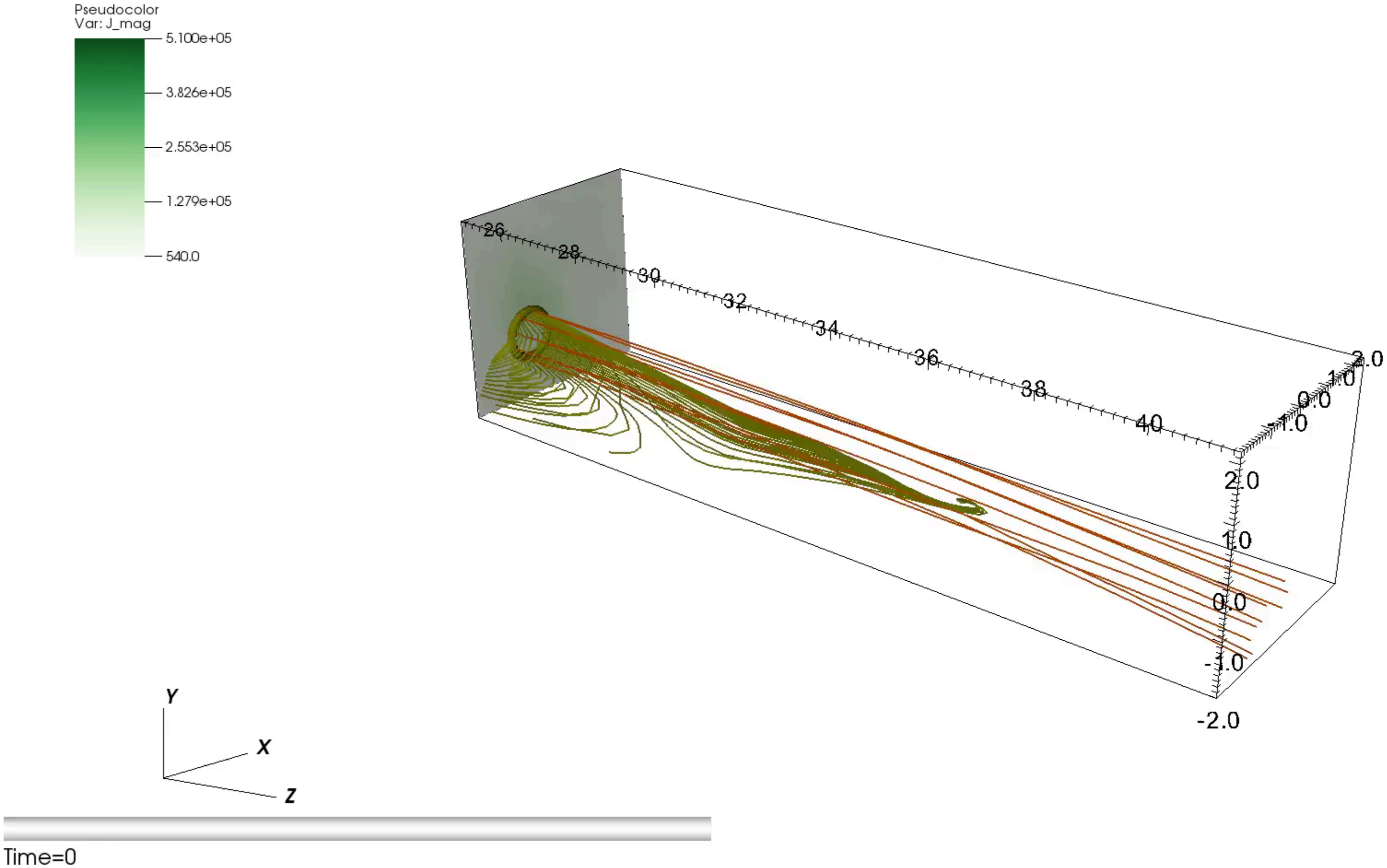
Reconstructed 3D Canonical Electron Flux Tubes

$$\int \vec{\Omega}_e \cdot d\vec{S} = \int \left(e\vec{B} + m_e\vec{\omega}_e\right) \cdot d\vec{S} \sim \int \vec{B} \cdot d\vec{S}$$



Reconstructed 3D Canonical Ion Flux Tubes

$$\int \vec{\Omega}_i \cdot d\vec{S} = \int \left(e\vec{B} + m_i \vec{\omega}_i\right) \cdot d\vec{S}$$



Ongoing Work: Constraining Ion Flow

Mach measurements incomplete, RSX is decommissioned

u_{iz} is measured in 2nd and 4th plane, u_{iy} is measured in 4th plane. Need to constrain u_{ix} and extrapolate $\vec{u}_i$ in 3D volume.

$\vec{J} = nq(u_i - u_e)$

Electrons frozen to magnetic field lines

$$\vec{u}_i \sim \frac{\vec{J}}{nq_e} + \alpha \frac{\vec{B}}{|B|}$$

Use the plane measurements of $\vec{u}_i$ to fit for $\alpha(x, y)$ and match the flux rope rotation as extrapolating along z.

Force Balance

Can the ion flows be extrapolated by balancing the centrifugal and Coriolis force terms balance the $\vec{J} \times \vec{B} - \nabla \mathrm{p}$?

Summary

Mochi.LabJet is designed to generate canonical flux tubes with skin and core currents, and axial and azimuthal shear flows.

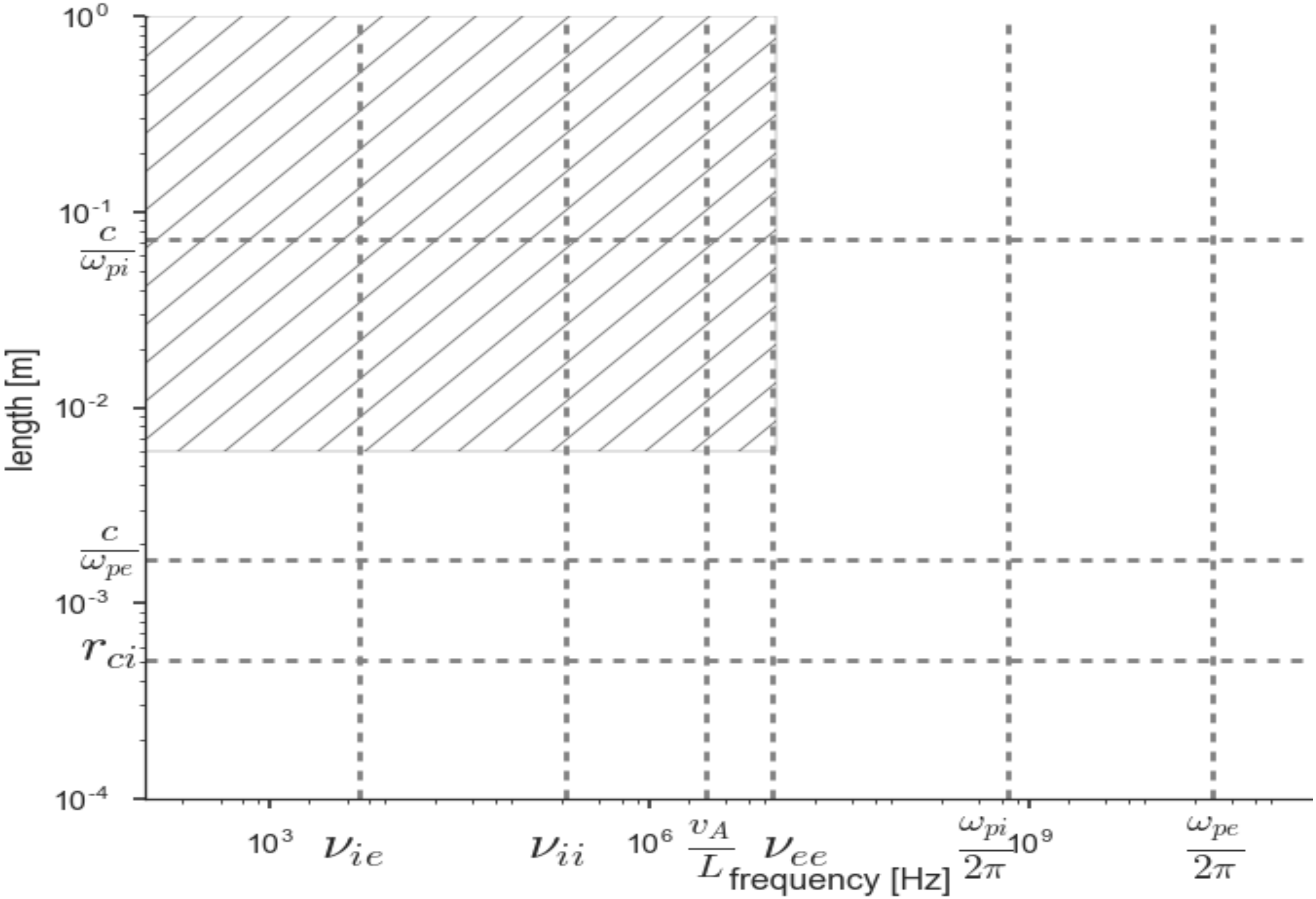
Analytical and numerical studies indicate that a lengthening flux tube may develop a sausage instability on top of a kink.

Reconstructing canonical flux tubes from magnetic field and ion flow measurements.

Acknowledgements

This work is supported by DOE Grant DE-SC0010340 and the DOE Office of Science Graduate Student Research Program and prepared in part by LLNL under Contract DE-AC52-07NA27344.

RSX diagnostic resolution



RSX Shot distributions